Substituting (3) for L in (2)

$$B_{12} = \frac{1}{\dfrac{\pi}{2\omega_1'}\left(\dfrac{\lambda_{g0}}{\lambda_0}\right)^2\left(\dfrac{f_2 - f_1}{f_0}\right)\dfrac{1}{\sqrt{C_1 L_2}}}. \tag{4}$$

Combining (1) and (4):

$$B_{12} = \frac{1}{\dfrac{\pi}{2}\left(\dfrac{\lambda_{g0}}{\lambda_0}\right)^2 K_{12}}. \tag{5}$$

In the general case:

$$B_{ij} = \frac{1}{\dfrac{\pi}{2}\left(\dfrac{\lambda_{g0}}{\lambda_0}\right)^2 K_{ij}} \tag{6}$$

where

B_{ij} = normalized susceptance of coupling circuit element between ith and jth resonator

K_{ij} = coefficient of coupling between ith and jth resonator.

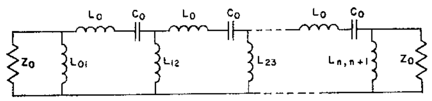

Fig. 2—Lumped circuit band-pass equivalent.

Eq. (6) can be employed to interchange normalized susceptances and coefficients of couplings for narrow-band direct-coupled waveguide band-pass filters. With appropriate modifications, this interchangeability can be extended to narrow-band lumped-circuit and coaxial band-pass filters.

It should also be noted that this paper is limited to interstage couplings. Input/output couplings have not been considered and will require a somewhat different analytical development.

Eq. (6) was independently derived and used by Sleven.[4]

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[4] R. L. Sleven, "Design of a tunable multi-cavity waveguide band-pass filter," 1959 IRE NATIONAL CONVENTION RECORD, pt. 3, pp. 91–112.

Contraphaseshifter*

SUMMARY

A novel combination of ring hybrid and variable power divider is shown to result in a three-terminal device having a single matched input and two outputs whose relative phase can be continuously varied independent of their amplitude.

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INTRODUCTION

A contraphaseshifter is defined here to mean a device which first splits a signal into two parts and then provides a means by which the relative phase between the two output terminals can be varied. Such a device has obvious application to linear-phased arrays for which the relative phase between corresponding elements on opposite sides of the center of the array must be varied in order to scan the beam. A contraphaseshifter is often comprised of a number of simpler devices. For example, a fixed two-way power divider feeding a pair of mechanically ganged variable phase shifters driven in opposite directions may be regarded as a contraphaseshifter. It is the purpose of this paper to describe a novel combination of two well-known microwave components which together constitute a contraphaseshifter even though neither component is a variable phase shifter per se.

THEORY OF OPERATION

Consider the ring hybrid shown in Fig. 1(a). An incident voltage of magnitude V at terminal 1 produces outputs at terminals 2 and 3 given by $V_2 = V_3 = 0.707V$. Since terminals 1 and 4 are isolated from each other, we may also apply simultaneously to terminal 4 a second voltage of magnitude V' which we will take to be in quadrature with V. For this condition the voltages at 2 and 3 become

$$V_2' = j0.707V' \quad \text{and} \quad V_3' = -j0.707V'.$$

The total voltages at 2 and 3 are

$$\begin{aligned} V_{2T} &= V_2 + V_2' \\ &= 0.707\sqrt{V^2 + V'^2}\, e^{j\tan^{-1} V'/V} \\ V_{3T} &= V_3 + V_3' \\ &= 0.707\sqrt{V^2 + V'^2}\, e^{-j\tan^{-1} V'/V}. \end{aligned}$$

Thus, the magnitudes of the voltages at 2 and 3 are equal to each other and dependent only on the sum of the squares of the voltages applied to 1 and 4, that is, dependent only on the total power into the ring hybrid. The relative phase of the voltages at 2 and 3, however, is dependent upon the ratio of V'/V. Thus:

$$\frac{V_{2T}}{V_{3T}} = e^{j2\tan^{-1} V'/V}. \tag{1}$$

To make a contraphaseshifter then, a second device is required which will

1) Split a signal into two quadrature components
2) Vary the magnitude of these components without varying their phase

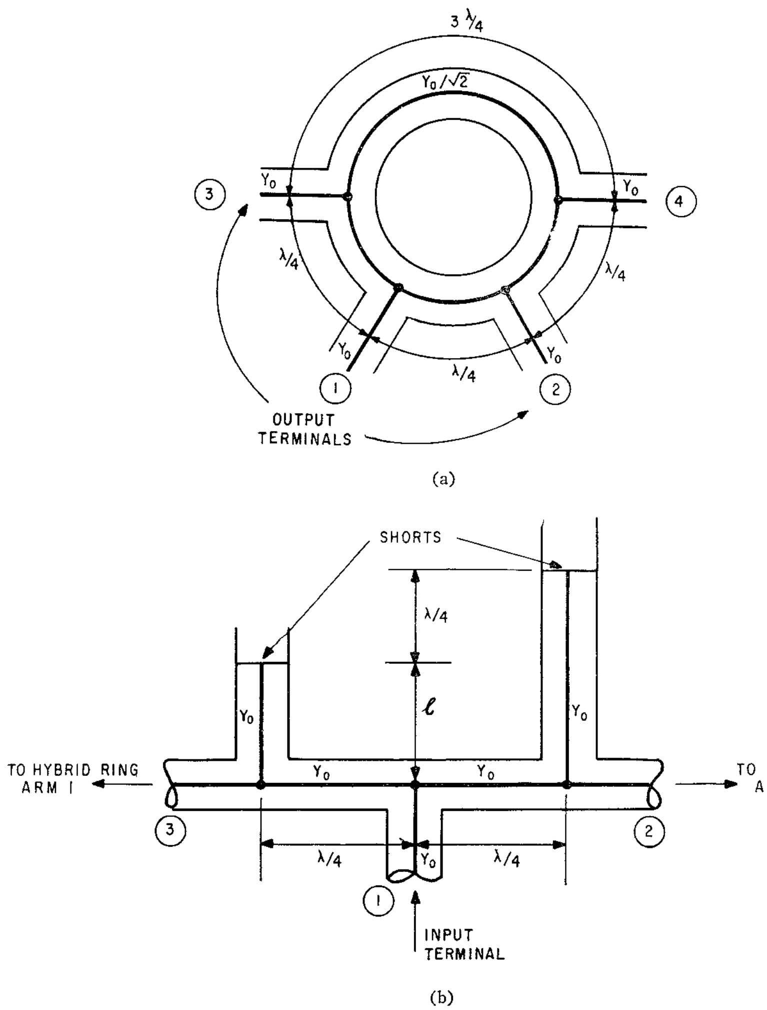

Fig. 1—Contraphaseshifter. (a) Hybrid ring. (b) Variable power divider.

3) Always present a match to the signal generator such that the total power into the ring hybrid remains constant.

Referring to the variable power divider shown in Fig. 1(b),[1] and using the standard transmission line equations,[2] the voltages at terminals 1, 2, and 3 are related by

$$V_1 = jV_2 \frac{Y_2}{Y_0} = jV_3 \frac{Y_3}{Y_0} \tag{2}$$

where Y_2/Y_0 and Y_3/Y_0 are the normalized loads on arms 2 and 3, namely, matched external loads plus the susceptances of the short-circuited shunt arms at 2 and 3. Thus:

$$\begin{aligned} \frac{Y_2}{Y_0} &= 1 + j \tan\left(\frac{2\pi l}{\lambda} + \frac{\pi}{2}\right) \\ &= 1 - j \operatorname{ctn} \frac{2\pi l}{\lambda} \\ \frac{Y_3}{Y_0} &= 1 + j \tan \frac{2\pi l}{\lambda} \cdot \end{aligned} \tag{3}$$

Combining (2) and (3), we obtain

$$\frac{V_2}{V_3} = \frac{Y_3}{Y_2} = \frac{1 + j \tan \frac{2\pi l}{\lambda}}{1 - j \operatorname{ctn} \frac{2\pi l}{\lambda}} = j \tan \frac{2\pi l}{\lambda} \cdot \tag{4}$$

If power divider arms 2 and 3 are connected to terminals 4 and 1 of the ring hybrid, then V'/V is equal to $\tan 2\pi l/\lambda$ and (1) may be written as

$$\frac{V_{2T}}{V_{3T}} = e^{j(4\pi l/\lambda)}. \tag{5}$$

Thus, the relative phase between outputs 2 and 3 of the ring hybrid varies linearly with the variable length l of the power divider. Note that the phase change produced is twice the change in electrical length of l. To prove that the power input to the ring hybrid remains constant, we will calculate the input admittance at terminal 1 of the power divider and show that it is invariant with l. The total current at terminal 1 is the sum of the currents I_{12} and I_{13} flowing toward loads 2 and 3 respectively. Using the transmission line equations,[3] we can write

$$\begin{aligned} I_{12} &= jV_2Y_0 \\ I_{13} &= jV_3Y_0. \end{aligned} \tag{6}$$

Adding (6) and substituting (2) and (3), we have

$$\begin{aligned} I_1 &= I_{12} + I_{13} = jY_0(V_2 + V_3) \\ &= Y_0V_1\left(\frac{Y_0}{Y_2} + \frac{Y_0}{Y_3}\right) \end{aligned}$$

$$Y_1 = \frac{I_1}{V_1} = Y_0 \left\{ \frac{1}{1 - j \operatorname{ctn} \frac{2\pi l}{\lambda}} + \frac{1}{1 + j \tan \frac{2\pi l}{\lambda}} \right\} = Y_0.$$

Hence, the admittance at input terminal 1 of the power divider is independent of l and matched to the line.

Equivalent forms of the devices shown in Fig. 1 are realizable in waveguide, coax, stripline, and lumped circuits. The technique, therefore, is applicable over a wide range of frequencies. Not only is the number of separate components reduced relative to the conventional fixed power divider dual phase shifter approach previously mentioned, but the technique should also be substantially more compact from a mechanical point of view relative to ganged and oppositely driven dual phase shifters.

Experimental Results

A variable power divider was constructed of rigid coax and coupled to a stripline hybrid via equilength flexible cables at 2300 Mc. Although the ideal conditions assumed in the analysis were not fully realized, a phase deviation of less than $\pm 10°$ and an amplitude variation of less than 1 db from that predicted by (5) was observed over a range of 360°. In general, it is believed that the degree to which the measured performance will deviate from the ideal is largely determined by the power divider. It has been shown that the power divider voltage output ratio must vary according to the function $\tan 2\pi l/\lambda$. In a practical situation the instantaneous 180° phase change of the tangent function as l goes through $\lambda/4$ is only a gradual one. Somewhat compensating for this gradual change in phase is the fact that practically all of the signal emerges from only one arm of the power divider over this range of l. Work on the power divider should produce the most fruitful results in future development of this technique.

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[1] G. L. Ragan, Ed., "Microwave Transmission Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 15, p. 519, 1948.
[2] H. P. Westman, Ed., "Reference Data for Radio Engineers," ITT Handbook, Stratford Press, Inc., New York, N. Y. 4th ed., p. 556; 1957.
[3] *Ibid.*, p. 557.

A Beam Plasma Surface Wave Interaction*

This communication describes the interaction of an electron beam with a beam-generated hydrogen plasma. As a result of this interaction the dc energy of the electron beam is converted into microwave energy in the plasma.

The experimental system is shown in Fig. 1. Coupling helices with essentially zero-db gain are located on both sides of an interaction cavity. An immersed gun of perveance 10^{-6} is used, and beam voltages are of the order of 660 volts. An axial magnetic field in the range of 500–1000 gauss focuses the beam. The system is pumped out, and gas is introduced by heating a titanium hydride capsule; this increases the pressure to the vicinity of 10^{-2} mm Hg. Beam electron collisions then ionize many of these hydrogen atoms, creating an electron density of approximately 10^{11} electrons/cm^3. As a result of the beam collision process, the plasma exists only in the immediate vicinity of the beam, so that the model which is assumed is that of a plasma column partially filling a cylindrical waveguide, with the plasma radius much smaller than the waveguide radius. The cross-section model is shown in Fig. 2.

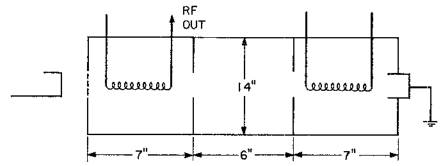

Fig. 1—Simplified experimental system.

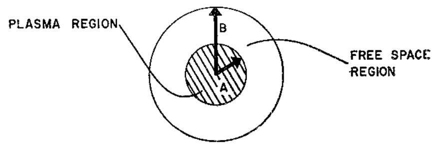

Fig. 2—Cross section of plasma column.

The results of the experiment are given by the following. With no signal applied to either helix, an RF signal higher than the cyclotron frequency is observed on the first helix as shown in Fig. 1. The signal, however, is not observed on the second helix. This signal is in the kilomegacycle region, and has a bandwidth of approximately 10 Mc. Its power was measured to be as high as 720 Mw, which, on the basis of 11-w input beam power, represents an efficiency of 6.5 per cent. Signals of a much lower power level were observed 80 Mc on each side of the center frequency; these sidebands were interpreted to be associated with ion plasma oscillations. By multiplying the ion plasma frequency by the square root of the mass ratio of electrons and ions, the electron plasma frequency can be computed to be 3200 Mc.

The plasma mode with which the beam interacts is identified as the angularly symmetric forward wave mode discussed by Gould and Trivelpiece[1] and Smullin and Chorney.[2] The ω-β curve of this mode for the case of $\omega_p > \omega_c$ is shown in Fig. 3. By intersecting the velocity line of the beam ($v/c = 0.05$) with the plasma ω-β curve for actual operating values of ω_c and ω_p, it is found that the intersection occurs at a frequency which is, in fact, the observed output frequency. (This process is actually per-

* Received April 23, 1962. This work was performed at Sperry Gyroscope Co. under Contract AF 33(616)7278, sponsored by Electronic Components Lab., Aeronautical Systems Div., Wright Field, Dayton, Ohio.

[1] R. W. Gould and A. W. Trivelpiece, "A new mode of wave propagation on electron beams," *Proc. Symp. on Electronic Waveguides*, Polytechnic Press, Polytechnic Inst. of Brooklyn, Interscience Publishers, N. Y., vol. 8 pp. 215–228; April, 1958.
[2] L. D. Smullin and P. Chorney, "Propagation in ion loaded waveguides," *Proc. Symp. on Electronic Waveguides*, Polytechnic Press, Polytechnic Inst. of Brooklyn, Interscience Publishers, N. Y., vol. 8, pp. 229–247; April, 1958.